Advanced Mathematical Models & Applications
Vol.6, No.1, 2021, pp.22-30

SOLITON SOLUTIONS OF HIROTA EQUATION AND HIROTA-MACCARI SYSTEM BY THE $\left(m+\frac{1}{G'}\right)$-EXPANSION METHOD

Hasan Bulut*, **Ayse Nur Akkilic**, **Ban Jamal Khalid**

Firat University, Elazığ, Turkey

Abstract. In this paper, the $\left(m+\frac{1}{G'}\right)$-expansion method is presented to seek the exact wave solutions of some nonlinear partial differential equations (NLPDEs), namely, the Hirota equation and the Hirota-Maccari system. The obtained solutions are solitary, topological, singular solitons and exponential function solutions. The 3D and 2D surfaces are also plotted for obtained solutions. This method is powerful, effective and it can be extended to many NLPDEs.

Keywords: The $\left(m+\frac{1}{G'}\right)$-expansion method, nonlinear partial differential equation, exact wave solutions, the Hirota equation, the Hirota-Maccari system.

AMS Subject Classification: 35E99, 35C07, 35C08, 35Q99.

Corresponding author: Hasan Bulut, Firat University, Elazığ, Turkey, e-mail: *hbulut@firat.edu.tr*

Received: 29 December 2020; Revised: 09 February 2021; Accepted: 02 March 2021; Published: 19 April 2021.

1 Introduction

In science, many important phenomena can be described by nonlinear partial differential equations. Seeking the exact solutions for these equations plays an important role in the study on the dynamics of those phenomena which appear in various scientific and engineering fields, such as solid-state physics, fluid mechanics, chemical kinetics, plasma physics, population models, and nonlinear optics (Gray & Scott, 1990; Ablowitz & Clarkson, 1991; Vakhnenko et al., 2003; El-Borai, 2008; El-Borai et al., 2011; Zayed & Arnous, 2012b; Hirota, 1980; Wang & Li, 2005; Wang & Zhang, 2005; Jawad et al., 2010; Zayed & Arnous, 2012a; Wang et al., 2008; Liu, 2006; Mirzazadeh et al., 2015a,b; Ismael et al., 2020; El-Borai et al., 2016; Demiray et al., 2016; Eslami et al., 2015; Yokus et al., 2018; Bulut & Khalid, 2020; Yokus et al., 2020; Ali et al., 2020). Many powerful methods have been proposed to obtain exact and approximated solutions of these models such as inverse scattering method Ablowitz & Clarkson (1991); Vakhnenko et al. (2003), Hirota bilinear transformation Hirota (1980), the f-expansion method Zayed & Arnous (2012a); Wang et al. (2008); Liu (2006), the modified simple equation method Jawad et al. (2010); Zayed & Arnous (2012a), the (G'/G)-expansion method Wang et al. (2008); Yokus et al. (2020), the $(1/G')$-expansion method Ali et al. (2020), the trial equation method Liu (2006); Mirzazadeh et al. (2015a) and the $\left(m+\frac{G'}{G}\right)$ $-$expansion method Ismael et al. (2020). In the present work, we use the $\left(m+\frac{1}{G'}\right)$-expansion method method for seeking the exact solutions for two important physical models, firstly the Hirota equation is given by, El-Borai et al. (2016)

$$iu_t + u_{xx} + 2|u|^2u + i\alpha u_{xxx} + 6i\alpha|u|^2u_x = 0. \tag{1}$$

which describe the propagation of the femto-second soliton pulse in the single-mode fibers, where $u = u(x,t)$ is the complex amplitude of the slowly varying optical field, the subscripts t and x indicate the temporal and spatial partial derivatives and α is small parameter.

Secondly, the Hirota-Maccari System is given by Demiray et al. (2016); Eslami et al. (2015)

$$\begin{aligned} &iu_t + u_{xy} + iu_{xxx} + uv - i|u|^2 u_x = 0, \\ &3v_x + \left(|u|^2\right)_y = 0. \end{aligned} \tag{2}$$

where $u = u(x,y,t)$ and $v = v(x,y,t)$ represent the complex scalar field and the real scalar field, respectively, while t represents the temporal variable x, y are the independent spatial variables.

2 The $\left(m + \frac{1}{G'}\right)$-expansion Method

Consider the general form of NPDEs (Nonlinear partial differential equations) as

$$P(u_{yy}, u_x u_{xz}, u_{xt}, u_{xxxz}, ...) = 0. \tag{3}$$

and using wave transformation

$$\phi(x,y,z,t) = U(\xi), \ \xi = c_1 x + c_2 y + c_3 z + c_4 t, \tag{4}$$

where $c_i \neq 0, \ (i = 1,2,3,4)$. Using Eq. (4) to Eq. (3) yields a nonlinear ODE for $U(\xi)$

$$O\left(U'', U'U'', U^{(4)}, ...\right) = 0. \tag{5}$$

The solution of Eq. (5) is assumed to have the form

$$U(\xi) = \sum_{i=-n}^{n} a_i (m+F)^i = a_{-n}(m+F)^{-n} + ... + a_0 + a_1(m+F) + ... + a_n(m+F)^n, \tag{6}$$

where $a_i, (\ i = 0,1,...,n)$ are constants, m is nonzero constant. According to the balancing principle, we find the value of n. Let F is defined as below

$$F = \frac{1}{G'(\xi)}, \tag{7}$$

and $G' = G'(\xi)$ provides the following second order ODE

$$G'' + (\lambda + 2m\mu) G' + \mu = 0, \tag{8}$$

where λ and μ are constants to be determined after. Putting the Eq. (6) to Eq. (5) and using Eq. (7), then collect all terms with the same order of the $(m+F)^n$, we get the system algebraic equations for $c_i \neq 0, \ (i = 1,2,3,4), \ a_i, (\ i = 0,1,...,n), \ \mu$ and λ.

As a result, we solve the obtained system to find the value of $c_i \neq 0, \ (i = 1,2,3,4)$ and $a_i, (\ i = 0,1,...,n)$ and inserting them into Eq. (6), we can study the explicit and exact solution of Eq. (3).

3 Applications of the $\left(m + \frac{1}{G'}\right)$-expansion Method

In this section, we implement the $\left(m + \dfrac{1}{G'}\right)$-expansion method to solve the presented models:

3.1 *Hirota Equation*

Putting the wave transformation

$$u(x,t) = U(\xi)e^{i\theta}, \xi = x + wt, \theta = px + qt. \tag{9}$$

we get the following two ordinary differential equations, respectively, real part and imaginary part of Eq. (1):

$$(p^3\alpha - p^2 - q)U(\xi) + (1 - 3\alpha p)U''(\xi) + (2 - 6\alpha p)U^3(\xi) = 0, \tag{10}$$

$$(w + 2p - 3\alpha p^2)U'(\xi) + \alpha U'''(\xi) + 6\alpha U^2(\xi)U'(\xi) = 0, \tag{11}$$

and constraint condition is

$$w = \frac{\alpha(p^3\alpha - p^2 - q)}{1 - 3\alpha p} + 3\alpha p^2 - 2p. \tag{12}$$

We can decide that

$$U''(\xi) + \left(\frac{p^3\alpha - p^2 - q}{1 - 3\alpha p}\right)U(\xi) + 2U^3(\xi) = 0. \tag{13}$$

Balancing between the terms U^3 and U'', we get $n = 1$. Later then considering Eq. (6) solutions,

$$U(\xi) = a_{-1}(m + F)^{-1} + a_0 + a_1(m + F)^1. \tag{14}$$

Substitution Eq. (14) into Eq. (13) by matching the coefficients $(m + 1/G')$ to zero, we get a system of equations. Solve the obtained system of equations, we can get the following cases of solutions.

Case 1. When we choose,

$$a_{-1} = \mathrm{i}m\,(\lambda + m\mu)\,, a_0 = -\frac{\mathrm{i}\lambda}{2}, a_1 = 0, \alpha = \frac{2p^2 + 2q + (\lambda + 2m\mu)^2}{p\left(2p^2 + 3(\lambda + 2m\mu)^2\right)}, \tag{15}$$

we get

$$u(x,t) = -\frac{1}{2}\mathrm{i}e^{\mathrm{i}(qt+px)}\left(\lambda - \frac{2m\,(\lambda + m\mu)}{m + \frac{1}{A_1K - \frac{\mu}{\lambda+2m\mu}}}\right),$$
$$K = \mathrm{e}^{-\frac{(\lambda+2m\mu)\left(4p^4t + 4p^3x + 6px(\lambda+2m\mu)^2 - 4p^2t\left(-3q + (\lambda+2m\mu)^2\right) + t(\lambda+2m\mu)^2\left(2q + (\lambda+2m\mu)^2\right)\right)}{2p\left(2p^2 + 3(\lambda+2m\mu)^2\right)}}. \tag{16}$$

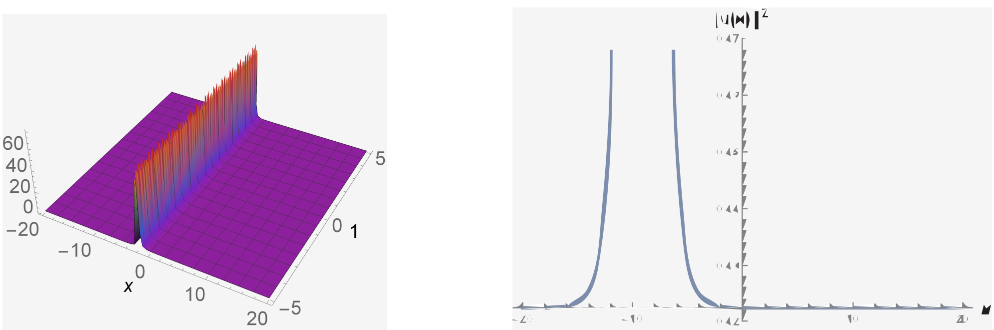

Figure 1: 3D surface and 2D surfaces of Eq. (16) when $A_1 = -1$, $p = 1.3$, $q = 0.5$, $m = 0.2$, $\lambda = 0.5$, $\mu = 1.5$ *and for* $t = 1$.

Case 2. When we choose,

$$a_{-1}=0, a_0=\frac{\mathrm{i}m\mu}{2}, a_1=-\mathrm{i}\mu, q=p^2\left(-1+p\alpha\right)+\frac{1}{2}m^2\left(-1+3p\alpha\right)\mu^2, \lambda=-m\mu \tag{17}$$

we get,

$$u(x,t)=\frac{\mathrm{i}e^{\frac{1}{2}\mathrm{i}\left(2p(x+pt(-1+p\alpha))+m^2t(-1+3p\alpha)\mu^2\right)}m\left(e^{m(x+pt(-2+3p\alpha))\mu+\frac{1}{2}m^3t\alpha\mu^3}+A_1m\right)\mu}{2\left(e^{m(x+pt(-2+3p\alpha))\mu+\frac{1}{2}m^3t\alpha\mu^3}-A_1m\right)}. \tag{18}$$

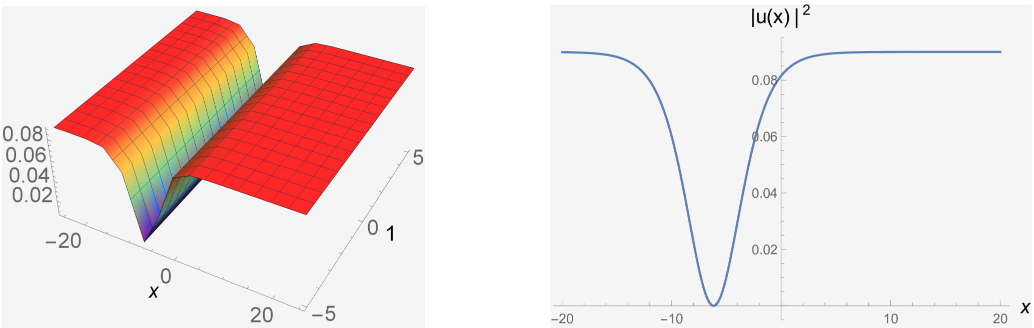

Figure 2: 3D surface and 2D surfaces of Eq. (18) when $A_1=-1,\ p=1.5,\ \alpha=1.2,\ m=0.6,$ $\mu=1,\ and\ for\ t=1.$

Case 3. When we choose,

$$a_{-1}=\frac{m\sqrt{-\left(\sqrt{q}-2m\mu\right)^2}\left(\sqrt{q}-m\mu\right)}{\sqrt{q}-2m\mu}, a_0=-\frac{1}{2}\sqrt{-(\sqrt{q}-2m\mu)^2}, a_1=0,$$
$$\lambda=\sqrt{q}-2m\mu, p=\mathrm{i}\sqrt{\frac{3}{2}}\sqrt{q}, \tag{19}$$

we get,

$$u(x,t)=\frac{1}{2}e^{\mathrm{i}qt-\sqrt{\frac{3}{2}}\sqrt{q}x}\sqrt{-(\sqrt{q}-2m\mu)^2}\left(-1+\frac{2m\left(\sqrt{q}-m\mu\right)}{\left(\sqrt{q}-2m\mu\right)\left(m+\frac{1}{A_1e^{\mathrm{i}\sqrt{6}qt-\sqrt{q}x+4q^{3/2}t\alpha-\frac{\mu}{\sqrt{q}}}}\right)}\right). \tag{20}$$

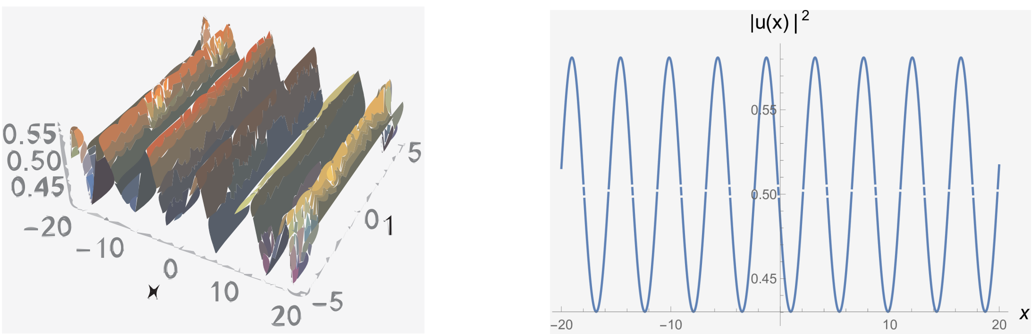

Figure 3: 3D surface and 2D surfaces of Eq. (20) when $A_1=-1.5,\ q=-2,\ m=0.025,\ \mu=1.08,$ $\alpha=2.5\ and\ for\ t=1.$

3.2 *Hirota-Maccari system*

Putting the wave transformation

$$u(x,y,t)=U(\xi)e^{i\theta}, v(x,y,t)=V(\xi), \xi=\delta(x+y-\kappa t), \theta=ax+by+rt, \tag{21}$$

into Eq. (6) and Eq. (7), the result yields the following nonlinear ordinary differential equation NODE

$$(3a-1)\,U^3+3\left(a^3-ab-r\right)U+3\delta^2\,(1-3a)\,U''=0,\ V+\frac{U^2}{3}=0, \tag{22}$$

and the constraint condition is

$$\kappa=\frac{(a+b-3a^2)(1-3a)-\left(a^3-ab-r\right)}{1-3a} \tag{23}$$

where a,b,r,δ are constants and $a\neq\frac{1}{3}$.

Balancing between the terms U^3 and U'' we get $n=1$. Using the value of balance, Eq. (6) become

$$U\left(\xi\right)=a_{-1}(m+F)^{-1}+a_0+a_1(m+F)^1. \tag{24}$$

Substitution Eq. (24) into Eq. (22) by matching the coefficients $(m+1/G'\ )$ to zero, we get a system of equations. Solve the obtained system of equations, we can get the following cases of solutions.

Case 1. When we choose,

$$a_0=\sqrt{\frac{3}{2}}\alpha\delta\lambda, a_1=\sqrt{6}\alpha\delta\mu, a_{-1}=0, b=\frac{2a^3-2r+3a\alpha^2\delta^2(\lambda+2m\mu)^2-\alpha\beta\delta^2(\lambda+2m\mu)^2}{2a}, \tag{25}$$

we get,

$$\begin{aligned}
&u\left(x,y,t\right)=\sqrt{\frac{3}{2}}\mathrm{e}^{\mathrm{i}(rt+ax+by)}\alpha\delta\left(\lambda+2\mu\left(m+K\right)\right),\\
&v\left(x,y,t\right)=-\frac{1}{2}\alpha\beta\delta^2(\lambda+2\mu\left(m+K\right))^2,\\
&K=\frac{\lambda+2m\mu}{-\mu+A_1\mathrm{e}^{\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+\frac{(a^3-ab-r)\alpha^2}{3a\alpha-\beta}+a\beta\right)\right)\delta(-\lambda-2m\mu)}(\lambda+2m\mu)}.
\end{aligned} \tag{26}$$

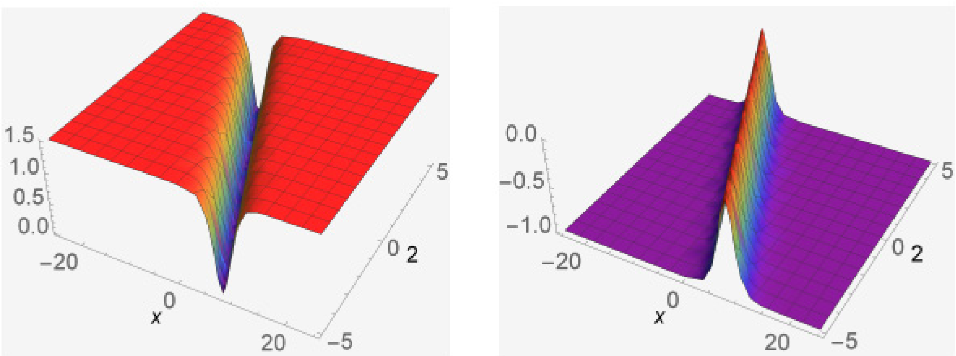

Figure 4: The 3D surfaces of Eq. (26) while $\delta=1, c=2, a=1, r=1,\ \lambda=3, \mu=-1,\ \kappa=1,\ \alpha=1,\ \beta=2,\ m=1, A_1=1$ *and for* $t=2$.

Case 2. When we choose,

$$a_0=0, a_1=-\frac{\sqrt{3}\sqrt{(-a^3+ab+r)\,\alpha}}{2\sqrt{m^2\,(3a\alpha-\beta)}}, a_{-1}=-\frac{\sqrt{3}m^2\sqrt{(-a^3+ab+r)\,\alpha}}{2\sqrt{m^2\,(3a\alpha-\beta)}},$$
$$\delta=\frac{\sqrt{-a^3+ab+r}}{2\sqrt{2}\sqrt{m^2\alpha\,(3a\alpha-\beta)\,\mu^2}}, \lambda=0, \tag{27}$$

we get,

$$u\,(x,y,t)=\mathrm{e}^{\mathrm{i}(rt+ax+by)}\left(-\frac{\sqrt{3}m^2\sqrt{(-a^3+ab+r)\,\alpha}}{B}-\frac{C}{2\sqrt{m^2\,(3a\alpha-\beta)}}\right)$$
$$B=2\sqrt{m^2\,(3a\alpha-\beta)}\left(m+\frac{2m\mu}{\left(-\mu+2A_1\mathrm{e}^{-\frac{m\sqrt{-a^3+ab+r}\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+a\beta-\frac{(a^3-ab-r)\alpha^2}{-3a\alpha+\beta}\right)\right)\mu}{\sqrt{2}\sqrt{m^2\alpha(3a\alpha-\beta)\mu^2}}}\right)m\mu}\right),$$
$$C=\sqrt{3}\sqrt{(-a^3+ab+r)\,\alpha}\left(m+\frac{2m\mu}{\left(-\mu+2A_1\mathrm{e}^{-\frac{m\sqrt{-a^3+ab+r}\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+a\beta-\frac{(a^3-ab-r)\alpha^2}{-3a\alpha+\beta}\right)\right)\mu}{\sqrt{2}\sqrt{m^2\alpha(3a\alpha-\beta)\mu^2}}}\right)m\mu}\right),$$
$$v\,(x,y,t)=\frac{\beta\left(-\frac{\sqrt{3}m^2\sqrt{(-a^3+ab+r)\alpha}}{D}-\frac{E}{2\sqrt{m^2(3a\alpha-\beta)}}\right)^2}{3\alpha},$$
$$D=2\sqrt{m^2\,(3a\alpha-\beta)}\left(m+\frac{2m\mu}{\left(-\mu+2A_1\mathrm{e}^{-\frac{m\sqrt{-a^3+ab+r}\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+a\beta-\frac{(a^3-ab-r)\alpha^2}{-3a\alpha+\beta}\right)\right)\mu}{\sqrt{2}\sqrt{m^2\alpha(3a\alpha-\beta)\mu^2}}}\right)m\mu}\right),$$
$$E=\sqrt{3}\sqrt{(-a^3+ab+r)\,\alpha}\left(m+\frac{2m\mu}{\left(-\mu+2A_1\mathrm{e}^{-\frac{m\sqrt{-a^3+ab+r}\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+a\beta-\frac{(a^3-ab-r)\alpha^2}{-3a\alpha+\beta}\right)\right)\mu}{\sqrt{2}\sqrt{m^2\alpha(3a\alpha-\beta)\mu^2}}}\right)m\mu}\right). \tag{28}$$

Case 3. When we choose,

$$a_0=\frac{\sqrt{3}\lambda\sqrt{\dfrac{a\left(-a^3+ab+r\right)\alpha\mu^2}{3a\alpha-\beta}}}{\mu\sqrt{a(\lambda+2m\mu)^2}}, a_1=\frac{2\sqrt{3}\sqrt{\frac{a(-a^3+ab+r)\alpha\mu^2}{3a\alpha-\beta}}}{\sqrt{a(\lambda+2m\mu)^2}}, a_{-1}=0,$$
$$\delta=\frac{\sqrt{2}\sqrt{-a^3+ab+r}}{\sqrt{\alpha\,(3a\alpha-\beta)\,(\lambda+2m\mu)^2}}, \tag{29}$$

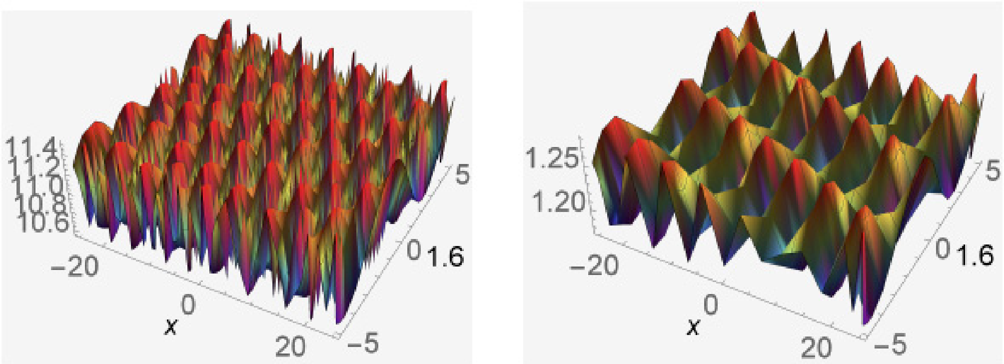

Figure 5: 3D graphs for Eq. (28) while $b = 1, a = 3.5, r = 2,\ \mu = 2,\ \alpha = 1.5,\ \beta = 0.5,$ $m = 4,\ A_1 = 1.2,\ and\ for\ t = 1.6.$

we get,

$$u(x,y,t) = \sqrt{3}e^{i(rt+ax+by)}\frac{\sqrt{-\frac{a(a^3-ab-r)\alpha\mu^2}{3a\alpha-\beta}}}{\sqrt{a(\lambda+2m\mu)^2}}\left(\frac{\lambda}{\mu}+2\left(m+\frac{\lambda+2m\mu}{-\mu+A_1 e^{\frac{\sqrt{2}\sqrt{-a^3+ab+r}\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+\frac{(a^3-ab-r)\alpha^2}{3a\alpha-\beta}+a\beta\right)\right)(-\lambda-2m\mu)}{\sqrt{\alpha(3a\alpha-\beta)(\lambda+2m\mu)^2}}}(\lambda+2m\mu)}\right)\right)$$

$$v(x,y,t) = \frac{\left(a^3-ab-r\right)\beta}{(3a\alpha-\beta)(\lambda+2m\mu)^2}\left(\lambda+2\mu\left(m+\frac{\lambda+2m\mu}{-\mu+A_1 e^{\frac{\sqrt{2}\sqrt{-a^3+ab+r}\left(x\alpha+y\beta-t\left(-3a^2\alpha+b\alpha+\frac{(a^3-ab-r)\alpha^2}{3a\alpha-\beta}+a\beta\right)\right)(-\lambda-2m\mu)}{\sqrt{\alpha(3a\alpha-\beta)(\lambda+2m\mu)^2}}}(\lambda+2m\mu)}\right)\right)^2. \tag{30}$$

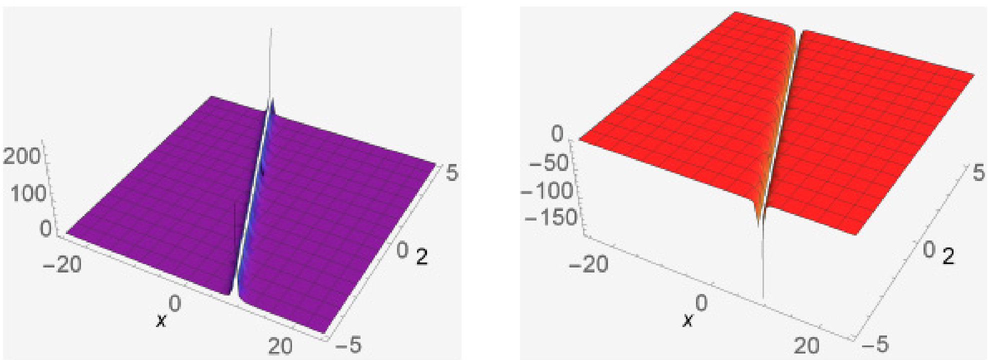

Figure 6: 3D graphs for Eq. (30) while $\delta = 1,\ c = 2,\ a = 1,\ r = 1,\ \lambda = 3,\ \mu = 1,\ \kappa = 1,$ $\alpha = 1,\ \beta = 2,\ m = 1,\ A_1 = 1,\ b = 1\ and\ for\ t = 2.$

4 Conclusion

In this paper, we successfully applied the proposed by the analytical method to solve the Hirota equation and the Hirota-Maccari system. As a result of these applications different types of solutions are obtained such as exponential function solutions, solitary, topological, and singular soliton solutions. The constraint conditions for the existence of these solutions are given. The 3D and 2D surfaces are also plotted for obtained solutions.

5 Acknowledgement

The authors would like to thank Azerbaijan University and the reviewers for their positive reports.

References

Ablowitz, M.J. & Clarkson, P.A. (1991) Soliton, *Nonlinear evolution equations and inverse scattering transform*, Cambridge University Press, New York .

Ali, K.K., Yilmazer, R., Yokus, A. & Bulut H. (2020). Analytical solutions for the $(3+1)-$dimensional nonlinear extended quantum Zakharov–Kuznetsov equation in plasma physics *Physica A: Statistical Mechanics and its Applications*, *548*, 124327.

Bulut, H. & Khalid, B.J. (2020). Optical Soliton Solutions of Fokas-Lenells Equation via $\left(m+\frac{1}{G'}\right)$-expansion Method. *Journal of Advances in Applied & Computational Mathematics*, *7*, 20-24.

Demiray, S.T., Pandir, Y. & Bulut, H. (2016). All exact travelling wave solutions of Hirota equation and Hirota-Maccari system *Optic*, *127*(4), 1848-1859.

El-Borai, M.M. (2008). Exact solutions for some nonlinear fractional parabolic fractional partial differential equations *Journal of Applied Mathematics and Computation*, *206*, 141-153.

El-Borai, M.M., Zaghrout, A.A. & Elshaer, A.L. (2011). Exact solutions for nonlinear partial differential equations by using the extended multiple Riccati equations expansion method *Inter. J. of Research and Reviews in Applied Sciences*, *9*(3),370-381.

El-Borai, M.M., El-Owaidy, H.M., Ahmed, H.M. & Arnous, A.H. (2016). Soliton solutions of Hirota equation and Hirota-Maccari system. *New Trends Math. Sci*, *4*(3), 231.

Eslami, M., Mirzazadeh, M.A., Neirameh, A.(2015). New exact wave solutions for Hirota equation. *Pramana J. of Physicst*, *84*(1), 3-8.

Gray, P. & Scott, S. (1990).*Chemical oscillations and instabilities*, Clarendon, Oxford.

Hirota R. (1980). *Direct method of finding exact solutions of nonlinear evolution equations, in: R. Bulloygh, P. Coudrey (Eds.), Bäcklund transformation* Springer, Berlin.

Ismael, H.F., Bulut, H. & Baskonus, H.M. (2010). Optical soliton solutions to the Fokas–Lenells equation via sine-Gordon expansion method and $\left(m+\frac{1}{G'}\right)$-expansion method, *Pramana*, *94*(1), 35.

Jawad, M.A.M., Petkovic, M.D. & Biswas, A. (2010). Modified simple equation method for nonlinear evolution equations. *Appl. Math. Comput.*, *217*(2), 869-877.

Liu, C.S. (2010). Trial equationmethod to nonlinear evolution equations with rank inhomogeneous: mathematical discussions and its applications. *Commun. Theor. Phys.*, *45*(2), 219-223.

Mirzazadeh, M., Arnous, A.H., Mahmood, M.F., Zerrad, E. & Biswas, A. (2015a). Soliton solutions to resonant nonlinear schrödingers equation with time-dependent coefficients by trial solution approach. *Nonlinear Dynamics*, *81*(1-2), 277-282.

Mirzazadeh, M., Arnous, A.H. & Eslami, M. (2015b). Dark optical solitons of Biswas-Milovic equation with dual-power law nonlinearity. *The European Physical Journal Plus*, *130*(4), 1-7.

Vakhnenko,V.O., Parkes, E.J. & Morrison, A.J. (2003). A Backlund transformation and the inverse scattering transform method for the generalized Vakhnenko equation. *Chaos, Solitons and Fractals*, *17*(4), 683-692.

Wang, M.L. & Li, X. (2005). Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation. *Chaos Solit. Fract.*, *24*(5), 1257-1268.

Wang, D.S. & Zhang, H.Q. (2005). Further improved F-expansion method and new exact solutions of Konopelchenko-Dubrovsky equations. *Chaos Solit. Fract.*, *25*(3), 601-610.

Wang, M.L., Li, X. & Zhang, J. (2008). The $\left(\frac{G'}{G}\right)$-expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics *Phys. Lett. A.*, *372*(4), 417-423.

Yokus, A., Sulaiman, T.A. & Bulut, H. (2018). On the analytical and numerical solutions of the Benjamin–Bona–Mahony equation. *Optical and Quantum Electronics*, *50*(1), 31.

Yokus, A., Durur, H., Abro, K.A. & Kaya, D. (2020). Role of Gilson–Pickering equation for the different types of soliton solutions: a nonlinear analysis. *The European Physical Journal Plus*, *135*(8), 657.

Zayed, E.M.E. & Arnous, A.H. (2012a). Exact solutions of the nonlinear ZK-MEWand the Potential YTSF equations using the modified simple equation method *AIP Conf. Proc*, *1479*, 2044-2048.

Zayed, E.M.E. & Arnous, A.H. (2012b). DNA dynamics studied using the homogeneous balance method *Chinese Physics Letters*, *29*(8), 080203.